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Mathematics and Its Ideologies (An Anthropologist's Observations)

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Abstract: Starting from the profound impact of Kenneth Arrow's Impossibility Theorem on the social sciences of the postwar twentieth century, this essay engages with the ways in which mathematics can be seen as a language-ideologically inflated notational system. In the mid-twentieth century, a profound belief in mathematics as a purely objective and non-ideological organization of knowledge took hold, and mathematical proof became the most authoritative type of statement on reality. When something was ruled 'logically impossible', real-world occurences could be seen as transgressions and exceptions. Hidden inside this belief is a set of irrational, metaphysical assumptions about humans and social behavior that can be laid bare by means of linguistic-anthropological analysis.

Keywords: ideology, mathematics, rational choice, literacy, notation, worldview

What is science? The question has been debated in tons of papers written over about two centuries, resulting in widely different views. Most people practicing science, consequently, prefer a rather prudent answer to the question, leaving some space for views of science that do not necessarily coincide with their own, but at least appear to share some of its basic features - the assumption, for instance, that knowledge is scientific when it has been constructed by means of methodologies that are shared intersubjectively by a community of scientific peers. The peer-group sharedness of such methodologies enables scientific knowledge to be circulated for critical inspection by these peers; and the use of such ratified methodologies and the principle of peer-group critique together form the "discipline" - the idea of science as *disciplined* knowledge construction.

There are, however, scientists who have no patience for such delicate musings and take a much narrower and more doctrinaire view of science and its limits. I already knew that - everyone, I suppose, has colleagues who believe that science is what *they* do, and that's it. But a small recent reading offensive on the broad social science tradition called Rational Choice (henceforth RC, also known as Rational Choice Theory) made me understand that such colleagues are only a minor nuisance compared to hardcore RC believers. For Rational Choice Theorists like Kenneth Arrow, William Riker, James M. Buchanan and their disciples, now spanning three generations, "scientific" equals "mathematical", period. Whatever is not expressed mathematically cannot be scientific; even worse, it is just "intuition", "metaphysics" or "normativity." And in that sense it is even *dangerous*: since "bad" science operates from such intuitive, metaphysical or normative assumptions, it sells ideology under the veil of objectivity and will open the door to totalitarian oppression. The following essay is a semiotic critique of this ideology of mathematics as used in RC.

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Sonja Amadae (2003), in a book I enjoyed reading, tells the story of how RC emerged out of Cold War concerns in the US. It was the RAND Corporation that sought, at the end of World War II and the beginning of the nuclear era, to create a new scientific paradigm that would satisfy two major ambitions. First, it should provide an objective, scientific grounding for decision-making in the nuclear era, when an ill-considered action by a soldier or a politician could provoke the end of the world as we knew it. Second, it should also provide a scientific basis for refuting the ideological ("scientific") foundations of communism, and so become the scientific bedrock for liberal capitalist democracy and the "proof" of its superiority. This meant nothing less than a new political science, one that had its basis in pure "rational" objectivity rather than in partisan, "irrational" a prioris. Mathematics rose to the challenge and would provide the answer.

Central to the problem facing those intent on constructing such a new political science was what Durkheim called "the social fact" - the fact that social phenomena cannot be reduced to individual actions, developments or concerns - or, converted into a political science jargon, the idea of the "public" or "masses" performing *collective* action driven by collective interests. This idea was of course central to Marxism, but also pervaded mainstream social and political science, including the (then largely US-based) Frankfurt School and the work of influential American thinkers such as Dewey. Doing away with it involved a shift in the fundamental imagery of human beings and social life, henceforth revolving around absolute (methodological) individualism and competitiveness modeled on economic transactions in a "free market" by people driven exclusively by self-interest. Amadae describes how this shift was partly driven by a desire for technocratic government performed by "a supposedly 'objective' technocratic elite" free from the whims and idiosyncrasies of elected officials (2003: 31). These technocrats should use abstract models - read *mathematical* models - of "systems analysis", and RAND did more than its share developing them. "Rational management" quickly became the key term in the newly reorganized US administration, and the term stood for the widespread use of abstract policy and decision-making models.

These models, as I said, involved a radically different image of humans and their social actions. The models, thus, did not just bring a new level of efficiency to policy making, they reformulated its ideological foundations. And Kenneth Arrow provided the key for that with his so-called "impossibility theorem", published in his *Social Choice and Individual Values* (1951; I use the 1963 edition in what follows). Arrow's theorem quickly became the basis for thousands of studies in various disciplines, and a weapon of mass political destruction used against the Cold War enemies of the West.

Arrow opens his book with a question about the two (in his view) fundamental modes of social choice: voting (for political decisions) and market transactions (for economic decisions). Both modes are seemingly *collective*, and thus opposed to dictatorship and cultural convention, where a single individual determines the choices. Single individuals, Arrow asserts, can be rational in their choices; but "[c]an such consistency be attributed to collective modes of choice, where the wills of many people are involved?" (1963:2). He announces that only *the formal aspects* of this issue will be discussed. But look what happens.

Using set-theoretical tools and starting from a hypothetical instance where two, then three perfectly rational individuals need to reach agreement, observing a number of criteria, he demonstrates that *logically*, such a rational collective agreement is impossible. Even more: in a smart and surely premeditated lexical move, in which one of Arrow’s criteria was “non-dictatorship” (i.e. no collective choice should be based on the preferences of one individual), Arrow demonstrated that the only possible “collective” choices would in fact be *dictatorial* ones. A political system, in other words, based on the notion of the common will or common good, would of necessity be a dictatorship. In the age of Joe Stalin, this message was hard to misunderstand.

And he elaborates this, then, in about hundred pages of prose, of which the following two fragments can be an illustration. (I shall provide them as visual images, because I am about ready to embark on my own little analysis, drawn from contemporary semiotic anthropology.)

From a logical point of view, some care has to be taken in defining the decision process since the choice of decision process in any given case is made by a decision process. There is no deep circularity here, however. If x is the vector describing a possible social state, let x_1 be the components of that vector which are not decision processes; let x_2 be the process of deciding among the alternative possible x_1's; in general, let x_n be the process of deciding among the alternative possible x_{n-1}'s. We may refer to x_1 as the first-order decision, x_2 as a second-order decision, etc.; then an nth-order decision is a process of choosing an $(n-1)$th-order decision method. Any particular social state is described in its entirety by a vector of the form $(x_1, x_2, \cdots, x_n, \cdots)$. In describing the United States Government, we might say that x_1 is a proposed bill or, more precisely, the proposed bill taken into conjunction with all the legislation now on the books; x_2 is the process by which bills are enacted into law by Congress and the President; x_3 is the process of choosing a Congress and President, set down by the Constitution; and x_4 is the process of constitutional amendment.

Figure 1. From Page 90, Arrow 1963

Now apply Lemma 6, replacing z by x and w by z; from $x\,R\,y$, which is assumed, follows $x\,R\,z$.

(b) $B(y, x, z)$: Suppose $y\,R_i\,z$ but not $x\,P_i\,z$. From the second statement follows $z\,R_i\,x$. From $y\,R_i\,z$ and $z\,R_i\,x$, we can conclude $y\,R_i\,x$. By the Assumption, replacing x by y and y by x, $y\,R_i\,x$ implies $x\,P_i\,z$, which contradicts the original supposition that both $y\,R_i\,z$ and not $x\,P_i\,z$ hold. Therefore, if $y\,R_i\,z$, then $x\,P_i\,z$. By Lemma 6, replacing x by y, y by z, z by x, and w by z, $x\,R\,z$ follows from $y\,R\,z$.

(c) $B(y, z, x)$: Suppose $y\,R_i\,z$. Then, by the Assumption, replacing x by y, y by z, and z by x, we can assert $z\,P_i\,x$. From $y\,R_i\,z$ and $z\,P_i\,x$, we have $y\,P_i\,x$. That is,

(5) $$y\,R_i\,z \text{ implies } y\,P_i\,x.$$

Let N' be the number of individuals for whom $y\,P_i\,x$, and N the number of individuals. Then, $x\,R_i\,y$ if and only if not $y\,P_i\,x$, so that

(6) $$N(x, y) = N - N'.$$

If $y\,P_i\,x$, then certainly $y\,R_i\,x$, so that

(7) $$N(y, x) \geq N'.$$

Since $x\,R\,y$, we have, by (1), that $N(x, y) \geq N(y, x)$; by (6) and (7), $N - N' \geq N'$, or

(8) $$N' \leq \frac{N}{2}.$$

From (5),

(9) $$N' \geq N(y, z).$$

For each i, either $y\,R_i\,z$ or $z\,R_i\,y$; therefore,

(10) $$N(y, z) + N(z, y) \geq N.$$

As $y\,R\,z$, we have, by (1), that $N(y, z) \geq N(z, y)$. By (10), this implies that $N(y, z) \geq N/2$. From (9), it follows that $N' \geq N/2$, and, with the aid of (8), $N' = N/2$. But this contradicts the hypothesis that the number of voters is odd. Hence, case (c) cannot arise; if $B(y, z, x)$, then we cannot have both $x\,R\,y$ and $y\,R\,z$.

Therefore, in every case where it was possible that $x\,R\,y$ and $y\,R\,z$, it was also true that $x\,R\,z$. R is transitive; this completes the proof of Theorem 4.

Figure 2. From Page 79, Arrow 1963

The prose on these pages became epochal: in it, one read the undeniable proof that collective rational social action was impossible, unless as a thinly veiled dictatorship - a death blow to Marxism of course, but also the definitive end of Durkheim's "social fact" - and that basing policy on such a collective rationality (as in welfare policy) was bound to be futile. This was now objectively, scientifically proven fact, established by the unimpeachable rigor of mathematical logic, which Arrow and his disciples believed that it could be applied to *any* aspect of reality.

Arrow, we saw, mentioned the limitations of his inquiry; evidently, he also used several assumptions. Amadae (2003: 84) lists four of them:

> "that science is objective; that it yields universal laws; that reason is not culturally relative; and that the individuals' preferences are both inviolable and incomparable."

The first three assumptions touch on his conception of science; in other words, they describe his belief in what mathematical methods do. I will return to them below. The fourth assumption is probably one of the most radical formulations of Methodological Individualism (henceforth MI). MI is the label attached to the theory complex in which every human activity is *in fine* reduced to individual interests, motives, concerns and decisions. In the case of Arrow and his followers, MI leads to the assumption that "society" is merely an aggregate of individuals. It is clear that this MI assumption - an *ideological* one, in fact a highly specific ideology of the nature of human beings and their social actions - underlies the "proof", makes it circular, and from an anthropological viewpoint frankly ridiculous, certainly when each of such individuals is a perfectly rational actor who

> "will always pursue his advantage, however he defines it, no matter what the circumstances; concepts of duty and responsibility are alien to the instrumental agent pursuing his goals" (Amadae 2003: 272)

Note that Arrow does not allow *comparison* between individuals (he will do so, grudgingly and conditionally, in 1977 in response to Rawls' discussion of justice: Amadae 2003: 270). This is important in three ways. One: it is a key motif in his "objective" approach, in which any normative judgment (e.g. a value judgment about preferences of individuals) needs to be excluded from the analysis, because any such judgment would bring in "irrational" elements and open the door to totalitarian policy options. Two: it thus underscores and constructs the case for mathematics as a method, about which more below. And three: it also provides a second-order ideological argument in favor of Man-the-individualist, for if individuals cannot be scientifically compared, they surely cannot be scientifically grouped into collectives.

And so, on the basis of a mathematical "proof" grounded in a set of highly questionable assumptions and operating on an entirely imaginary case, Arrow decided that society – the real one – is made up of a large number of individuals bearing remarkable similarities to Mr. Spock. And this, then, was seen as the definitive scientific argument against Marxism, against the Durkheimian social fact, against the welfare state, socialism and communism, and in favor of liberal democracy and free market economics. It is, carefully considered, a simple ideological propaganda treatise covered up by the visual emblems of mathematics-as-objective-science. The assumptions it takes on board as axiomatic givens constitute its ideological core, the mathematical “proof” its discourse, and both are dialectically interacting. His assumptions contain everything he claims to reject: they are profoundly normative, idealistic, and metaphysical. Every form of subjectivity becomes objective as long as it can be formulated mathematically.

The fact that his “impossibility theorem” is, till today, highly influential among people claiming to do *social* science, is mysterious, certainly given the limitations specified by Arrow himself and the questionable nature of the assumptions he used - the most questionable of which is that of universality, that mathematics could be used to say something sensible on the nature of humans and their societies. The fact that these people often also appear to firmly believe that Arrow’s formal *modeling* of social reality, with its multitude of Spocks, is a pretty accurate *description* of social reality, is perplexing, certainly knowing that this mathematical exercise was (and is) taken, along with its overtly ideological assumptions, to be simple social and political fact (observable or not). Notably the MI postulate of individuals behaving (or *being*) like entirely sovereign and unaffected consumers in a free market of political choices, "proven" by Arrow and turned into a factual (and normative) definition, leads Adamae (2003: 107) to conclude "that Arrow's set-theoretical definition of citizens' sovereignty is one of the least philosophically examined concepts in the history of political theory.” Nonetheless, this definition promptly and effectively eliminated a number of items from the purview of new political science: the public sphere, the common good, and even society as such – Arrovians would use the simple argument that since society was not human (read: not individual and rational), it could not be seen as an actor in its own right. Margaret Thatcher, decades later, agreed.

Arrow and his followers set new standards of political debate, arguing that political issues (think of social welfare) were not "real" if they didn't stand the test of logical analysis. Unless facts agreed with mathematical coherence (as shown in Fig. 2 above), they were not proven facts; mathematics became the standard for defining reality, and the phrase “theoretically impossible” became synonymous for impossible in reality, separating fact from fiction. I find this unbelievable. But the point becomes slightly more understandable when we broaden the discussion a bit and examine more closely the particular role of mathematics in all of this. And here, I turn to semiotic anthropology.

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My modest reading offensive also brought Izhtak Gilboa's Rational Choice (2010) to my table. Gilboa – a third-generation RC scholar with basic training in mathematics – offers us a view of what I prefer to see as the *ideology of mathematics* in all its splendor and naiveté. Before I review his opinions, I hasten to add that Gilboa is quite critical of radical interpretations of Arrovian choice, including Game Theory, admitting that the complexity of real cases often defies the elegance of established theory, and that we should "keep in mind that even theoretically, we don't have magic solutions" (2010: 85). Yet he declares himself a full blown adept of RC as a "paradigm, a system of thought, a way of organizing the world in our minds" (2010: 9). And this paradigm is encoded in mathematical language.

Gilboa expresses an unquestioned faith in mathematics, and he gives several reasons for this.

1. **Accuracy:** Mathematics is believed to afford the most accurate way of formulating arguments. "The more inaccurate our theories are, and the more we rely on intuition and qualitative arguments, the more important is mathematical analysis, which allows us to view theories in more than one way" (20). Theories not stated in mathematical terms, thus, are suggested *not* to allow more than one way of viewing.
2. **Rigor:** Mathematics brings order in the chaos. Such chaos is an effect of "intuitive reasoning" (29). Thus, mathematical formulations are rigorous, ordering modes of expressing elaborate conglomerates of facts, not prone to misunderstanding. They form the *theoretical* tools of research, bringing clear and unambiguous structure in fields of knowledge in ways not offered by "intuitive reasoning." The latter is a curious category term, frequently used by Gilboa to describe, essentially, any form of knowledge construction that cannot yet be expressed in mathematical language.
3. **Superiority.** This follows from (1) and (2). There is mathematics and there is the rest. The rest is muddled and merely serves to test the mathematical theory. Thus (and note the evolutionary discourse here, marked in italics), when a mathematical theoretical model is thrown into "more elaborate research", such research may prove to be "too complicated to carry out, and we will *only make do* with intuitive reasoning. In this case we try to focus on the insights that *we feel* we understand *well enough to be able to explain verbally*, and independently of the specific mathematical model we started out with" (29). Non-mathematically expressed knowledge is obviously inferior to mathematically expressed knowledge: it is "intuitive." Yet, it has its importance in theory testing: "mathematical analysis has to be followed by intuitive reasoning, which may sort out the robust insights from those that only hold under very specific assumptions" (ibid).

4. **Simplification:** throughout the entire book, but actually throughout most of what we see in RC at large, there is a marked preference for mathematically expressed simplicity. Complex real-world problems are reduced to extremely simple hypothetical cases involving pairs or triplets, as when complex market transactions are reduced to two people bargaining in a game-theoretical example, or the three Spocks in Arrow's Impossibility Theorem who are supposed to instantiate millions of voters or consumers in large-scale political and economic processes. Such mathematical simplifications often bear names – the Prisoners' Paradox, Condorcet's Paradox, the Pareto Optimality or the Von Neumann-Morgenstern Axioms – and are presented (be it with qualifications) as "laws" with universal validity. The simple cases of mathematical theory are proposed as accurate, rigorous and superior modes of describing (and predicting) complex realities.
5. **Psychological realism.** Not only are the mathematical formulations accurate descriptive and predictive models of actual social realities, they are also an accurate reflection of human cognitive folk methods, even if people are not aware of it: "Statistics is used to estimate probabilities explicitly in scientific and nonscientific studies as well as implicitly by most of us in everyday life" (56-57). Gilboa as well as many other authors doing this kind of work have the amusing habit of describing people who apply the Von Neumann-Morgenstern Axioms in deciding where to take their holidays and experience very severe logical problems when their behavior violates the Prisoners' Paradox or exhausts the limits of objective reasoning.
6. **Convincing-conclusive.** Finally, Gilboa makes a somewhat curious point about "positive" versus "negative rhetoric." Negative rhetoric consists of "tricks that make one lose the debate but for which one has good replies the morning after the debate", while "positive rhetoric consists of devices that you can take from the debate and later use to convince others of what you were convinced of yourself. Mathematics is such a device" (19).

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The six features of Gilboa's approach to mathematics are, I would argue, an *ideology of mathematics*. They articulate a socioculturally entrenched set of beliefs about mathematics as a scientific project. And while I am the first to express admiration for mathematics as a scientific tool which, indeed, allows a tremendous and unique parsimony, transparency and stability in notation, I think the broader ideology of mathematics needs to be put up for critical examination. For mathematics, here, is not presented as a scientific tool – call it "method" or even "methodology" – but as an *ontology*, a statement on how reality "really" is. We already encountered this earlier when I discussed the mystery of Arrow's Theorem: no facts are "real" unless they can be expressed in mathematical, formal language. And to this, I intend to attach some critical reflections.

Let me first describe the ontology I detect in views such as the ones expressed by Gilboa, occasionally returning to Arrow's first three assumptions mentioned earlier. I see two dimensions to it.

1. Mathematics expressions are *the Truth* since mathematics represents the perfect overlap of facts and knowledge of facts. And this Truth is *rationality*: mathematical expressions are expressions of fundamental rationality, devoid of all forms of subjectivity and context-dependence. This enables mathematical expressions to be called "laws", and to qualify such laws as eternal, universal, and expressions of extreme certainty and accuracy. Recall now Arrow's second and third assumption: that science (i.e. mathematics) yields universal laws, and that reason is not culturally relative – since it can be described in a universal mathematical code.
2. Mathematics as an ontology has both *esoteric* and *practical* dimensions, and these dimensions make it *science*. Concretely, mathematics is not something everyone can simply access because it is esoteric – see Fig 2 above for a graphic illustration – and it is practical because it can be applied, as a set of "laws" flawlessly describing and predicting states of reality, to a very broad range of concrete issues, always and everywhere.

Combined with the first point, mathematics as the (rational) Truth, we understand not just Arrow's first assumption – that science is objective – but his wider (political) project as well. The scientific underpinning of a new social and political science had to be mathematic, because that was the way to avoid ideological, cultural or other forms of "subjectivity" which would make such a science "irrational", and may lead it towards totalitarian realities. Mathematically stated laws (on *any* topic) are – so it is suggested – independent of the person formulating them or the place in the world from where they are formulated; their truth value is unconditional and unchallengeable; accepting them is not a matter of personal conviction or political preference, it is a matter of universal reason. This is why Gilboa found mathematics convincing and conclusive: confronted with mathematical proof, no reasonable person could deny the truth, for, as expressed by Gilboa, mathematical formulations reflected – esoterically – the folk reason present in every normal human being. And so we see the comprehensive nature of the ontology: mathematics *describes human beings* and by extension *social life* in a truthful, unchallengeable way.

It is at this last point – the "postulate of rationality" as it is known in RC – that this modern ideology of mathematics appears to have its foundations in Enlightenment beliefs about reason as fundamentally and universally human, and so deviates from older ideologies of mathematics. These are *well documented*, and there is no need here to review an extensive literature, for a key point running through this history is that mathematics was frequently presented as the search for the true and fundamental structure of nature, the universe and (if one was a believer) *God's creation*. This fundamental structure could be expressed in rigorous symbolic operations: specific shapes, proportions, figures and relations between figures – they were expressed by means of *abstract* symbols that created the esoteric dimension of mathematics. Doing mathematics was (and continues to be) often qualified as the equivalent of being "a scientist" or "a wise man", and if we remember *Newton*, the distinction between scientific mathematics and other esoteric occupations such as alchemy was often rather blurred.

It is in the age of Enlightenment that all human beings are defined as endowed with reason, and that mathematics can assume the posture of the science describing the fundamental features and structures of this uniquely human feature, as well as of the science that will push this unique human faculty forward. It is also from this period that the modern individual, as a concept, emerges, and the American Declaration of Independence is often seen as the birth certificate of this rational, sovereign individual. Emphasis on rationality very often walks hand in hand with methodological individualism, and this is not a coincidence.

Observe that this ideology of mathematics is pervasive, and even appears to be on the rise globally. Mathematics is, everywhere, an element of formal education, and universally seen as "difficult." Training in mathematics is presented in *policy papers* as well as in folk discourse as the necessary step-up towards demanding professions involving rigorous scientific reasoning, and success or failure in mathematics at school is often understood as an effect of the success/failure to enter, through mathematics, a "different mode of thinking" than that characterizing other subjects of learning. Mathematics, in short, often serves as a yardstick for appraising "intelligence."

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From the viewpoint of contemporary semiotic anthropology, this ideology of mathematics is just another, specific, *language ideology: a set of socioculturally embedded and entrenched beliefs attached to specific forms of language use.* The specific form of language use, in the case of mathematics, is a form of literacy, of writing and reading. So let us first look at that, keeping an eye on Figures 1 and 2 above.

Mathematics as we know it gradually developed over centuries as a separate *notation* *system* in which random symbols became systematic encoders of abstract concepts – quantities, volumes, relations. Hardcore believers will no doubt object to this, claiming that the notational aspect is just an "instrumental", ancillary aspect and that the core of mathematics is a special form of reasoning, a special kind of cognitive process. They are wrong, since the notational system is the very essence of the cognitive process claimed to be involved, which is why mathematicians *must* use the notational systems, and why school children can "understand" quite precisely what they are being told in mathematics classes but fail their tests when they are unable to convert this understanding into the correct notation. Seeing knowledge as *in se* detached from its infrastructures and methods of production and transmission is tantamount to declaring the latter irrelevant - which begs the question as to why mathematics uses (and insists on the use of) a separate notation system. More on this below.

The system, furthermore, is a socioculturally marked one, and the evidence for that should be entirely obvious. Recall Figure 2. The mathematical notation system follows the left-to-right *writing vector* of alphabetical scripts (not that, for instance, of Arabic or Chinese); unless I am very much mistaken "written" mathematical symbols (as opposed to e.g. geometrical figures) are *alphabetical* and not, e.g. hieroglyphic, cuneiform or ideographic (like Chinese characters); and they are drawn from a *limited number of alphabets*, notably Greek and Latin alphabets. Just click the "special symbols - mathematical symbols" icon in your word processor now for double-checking. In spite of historical influences from Ancient Egypt and Babylonia, the Arab world, India and China, 19th century codification and institutionalization of mathematics (like other sciences) involved the Europeanization of its conventions.

The system is *separate* in the sense that, in spite of its obvious origins, it cannot be reduced to the “ordinary” writing system of existing languages: the fact that the symbol "0" for "zero" is of Indian origins doesn't make that symbol Sanskrit, just as the Greek origins of the symbol for "pi" do not load this symbol with vernacular Greek meanings; they are *mathematical* symbols. But it can be *incorporated* (in principle) in any such writing system – Figures 1 and 2 show incorporation in English, for instance – and *translated*, if you wish, in the spoken varieties of any language (something it shares with Morse code). The symbol "<" for instance, can be translated in English as “less/smaller than.” Figure 1 above shows how Arrow translates ordinary English terms into mathematical terms, and the language-ideological assumption involved here is that this translation involves perfect denotational equivalence (the symbols mean exactly what the words express), as well as a superior level of accuracy and generalizability (the concrete of ordinary language becomes the abstract-theoretical of mathematical notation – the words become concepts). Here, we see what language ideologies are all about: they are a synergy of concrete language forms with beliefs about what they perform in the way of meaning. Thus, the difference between ordinary writing and mathematical writing is the belief we have that the latter signals abstraction, theory, and superior accuracy (something for which logical positivism provided ample motivational rhetoric).

This notation system is, in contemporary anthropological vocabulary, best seen as a *specialized graphic register*. That means that it can be used for a limited set of specific written expressions, as opposed to an "ordinary" writing system in which, in principle, anything can be expressed. We see it in action in the way I just described – reformulating ordinary expressions into "concepts" – in Figure 1, while Figure 2 shows that the register can be used for entire "textual" constructions in the genre of "proof." The register is parsimonious and, in that sense, efficient. Writing "125364" requires just six symbols; writing "one hundred and twenty-five thousand three hundred and sixty-four" demands almost ten times that number of symbols.

It is, as a graphic register, extremely *normative*; it is an "ortho-graphy." Mathematics deploys a closed and finite set of standardized symbols that have to be used in rigorously uniform ways – the symbol "<" can never mean "more than"; both their individual meaning and the ways in which they can be syntactically combined are subject to uniform and rigid rules. Consequently, while in "ordinary" writing some errors do not necessarily distort the meaning of an expression (most people would understand that "I cam home" means "I came home"), a writing error in mathematical notation renders the expression meaningless. So many of us painfully experienced this in the mathematics classes we took: our understanding of the fundamentals of mathematics did not include any degree of freedom in choosing the ways to write it up, since mathematics *is* normative, orthographic notation. This, too, is part of its specialized nature as well as of its esoteric nature: mathematics must be acquired through disciplined – nonrandom and highly regimented – learning procedures, and knowledge of specific bits of the register are identity-attributive. Some mathematicians are specialists of calculus, others of logic, for instance, while the identity label of "genius" would be stuck on outstanding mathematicians of any branch.

That is the specific form of language we see in mathematics; the language-ideological values attributed to it are, like any other language ideology, sociocultural constructs that emerged, are consolidated and develop by observing socioculturally ratified rules and procedures; and these are (like any other sociocultural convention) highly sensitive to developments over time and place. Very few contemporary mathematicians would be ready to defend the claim that mathematics reveals the fundamental structure of God's creation, for instance, but it is good to remember that this language-ideological value was once attached to it, and that the people who attached it to mathematics were profoundly convinced that this was what mathematics was all about. Similarly, not too many contemporary mathematicians would perceive alchemy as an occupation compatible with the scientific discipline of mathematics, while Isaac Newton appeared not to have too many doubts about that.

There is nothing eternal, absolute or undisputable to the language-ideological assumptions accompanying mathematics. The suggestion, as I noted a widespread one, that mathematics would involve a “different way of thinking” is a quite questionable one. It is a *different way of writing*, to which a specific set of language-ideological values are attached. Children who are “not good at mathematics” at school, probably have more of a literacy problem than of a cognitive one – let alone one of inferior intelligence.

And if we return to Gilboa's six features above, we might perhaps agree that his first two features – accuracy and rigor – are intrinsic affordances of the specific register of mathematics (things mathematics indeed can do quite well). The third feature (superiority) is a belief probably shared by members of the community of mathematicians, but not *per se* demonstrable, quite the contrary. Because the fourth feature – simplification – points to a *limitation* of the register, i.e. the fact that not everything can be appropriately written in the code. Ordinary language writing offers an infinitely vaster set of affordances. It is, at this point, good to remind ourselves of the fact that abstraction involves "stripping down", i.e. the *deletion of features* from a chunk of reality; that this deletion may touch *essential* features; and that this deletion is often done on the basis of unproven assumptions.

The fifth feature – psychological realism – cries out for evidence, and those familiar with (to name just one) Alexander Luria's 1920s research on modes of thought will be inclined to take a more sobering and prudent view on this topic. There is no reason why the fundamental structures of rationality would not be expressed, for example, in narrative-poetic patterns rather than in mathematical-logical ones. And as for the sixth feature – the conclusive nature of mathematical proof: this, I suppose, depends on whom one submits it to. If the addressee of a mathematical argument shares the ideological assumption that such an argument is conclusive, s/he will accept it; if not, submitting mathematical proof might be not more conclusive than singing a Dean

Martin song.

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Language-ideological attributions are always sociocultural constructs, and therefore they are never unchallengeable and they can always be deconstructed. What we believe certain forms of language do, does not necessarily correspond to what they effectively do. There is, for example, a quite widely shared language-ideological assumption that grammatical, orthographic or other forms of "correctness" are strict conditions for understandability ("you can only make yourself understood if you speak standard language!"), while realities of human interaction show that tremendous largesse often prevails, without impeding relatively smooth mutual understanding. There is also a widespread language-ideological belief that societies are monolingual (think of the official languages specified in national legislations and, e.g., adopted by the EU), while in actual fact dozens of languages are being used. It is the job of my kind of anthropologists and sociolinguists to identify the gaps between facts and beliefs in this field.

Seen from that perspective, there is nothing *in se* that makes a mathematical proof more "objective" than, say, a poem or a newspaper article. The status of "objectivity", indeed the very *meaning* of that term, emerges by sociocultural agreement within specific communities, and none of the features of the register are in themselves and directly elements of "objectivity." The notion of objectivity as well as the symbols that are proposed as "indexes" of objectivity, are all sociocultural constructs.

Paradoxically, thus, if we recall Kenneth Arrow's extraordinarily far-reaching claims, the status of objectivity attributed to mathematics is a vintage Durkheimian social fact: something produced as a norm by societies and accepted by individuals for reasons they themselves often ignore – it's a sociocultural convention wrapped, over time, in institutional infrastructures perpetuating and enforcing the convention (in the case of mathematics, the education system plays first violin here). Its power – *hegemony* we would say – does not turn it into an *absolute* fact. It remains perpetually challengeable, dynamic, an object of controversy and contention as well as a proposition that can be verified and falsified. Saying this is nothing more than stating the critical principles of science as an Enlightenment product, of re-search as literally meaning "search again" even if you believe you have discovered the laws of nature. These critical principles, we will recall, were the weapons used against religious and dictatorial ("irrational") postures towards the Truth. They are the very spirit of science and the engine behind the development of sciences.

The intimate union between RC, mathematics, MI and the specific views of human nature and social action that were articulated in this movement, cannot escape this critique. Practitioners of this kind of science would do well to keep in mind that a very great number of their assumptions, claims and findings are, from the viewpoint of other disciplines involved in the study of humans and their societies, simply absurd or ridiculous. The axiomatic nature of rationality, the impossibility of collective choice and action, the preference for extraordinarily pessimistic views of human beings as potential traitors, thieves and opportunists – to name just these – are contradicted by mountains of evidence, and no amount of deductive theorizing can escape the falsifications entailed by this (inductive and not at all, *pace* Gilboa, "intuitive") evidence.

MI, leading, as in Arrow's work, to the refusal to compare individuals' preferences and to isolate human beings from the complex patterns of interaction that make up their lives, is simply ludicrous when we consider, for instance, language – a system of shared normatively organized sociocultural codes (a "social fact", once more) which is rather hard to delete from any consideration of what it is to be human or dismiss as a detail in human existence. Here we see how the "stripping down" involved in mathematical abstraction touches essential features of the object of inquiry, making it entirely unrealistic. We have also seen that the language in which such "truths" are expressed is, in itself, a pretty obvious falsification of MI and other RC assumptions. And more generally, facts gathered through modes of science that Gilboa tartly qualifies as "intuitive reasoning" are also always *evidence of something*, and usually not of what is claimed to be true in RC.

Such critiques have, of course, been brought to RC scholars (an important example is Green & Shapiro 1994). They were often answered with definitional acrobatics in which, for instance, the concept of "rationality" was stretched to the point where it included almost everything, so as to save the theory (but of course, when a term is supposed to mean *everything* it effectively means *nothing*). Other responses included unbearably complex operations, attempting to keep the core theory intact while, like someone extending his/her house on a small budget, adding makeshift annexes, windows, rooms and floors to it, so as to cope with the flurry of exceptions and unsolvable complexities raised against it. I found, for instance, Lindenberg's "method of decreasing abstraction" (1992) particularly entertaining. Recognizing the complexity of real-world issues, and aiming (as anyone should) at *realism* in his science, Lindenberg constructs a terrifically Byzantine theoretical compound in which the scientist gradually moves away from simple and rigid mathematical formulations towards less formal and more variable formulations – hence "decreasing abstraction" or "increasing closeness to reality" (Lindenberg 1992: 3). He thus achieves, through admirably laborious theoretical devotion, what any competent anthropologist achieves in his/her fieldnotes at the end of a good day of ethnographic fieldwork.

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This brings me to a final point. Mathematics is a formal system, and a peripheral language-ideological attribution it carries is that of "theory." Theory, many people believe, *must* be abstract, and whatever is abstract must be theoretical. People working in the RC paradigm like to believe that (theoretical) "generalization" in science can only be done through, and is synonymous with, abstraction – mathematical expression in formulas, theorems, or statistical recipes.

In dialogues with such people, it took me a while before I detected the cause of the perpetual misunderstandings whenever we tried to talk about issues of generalization and theorization across disciplines, for we were using the same words but attached them to entirely different cultures of interpretation. Their culture was usually a deductive one, in which theory came first and facts followed, while mine operated precisely the other way around. I had to remind them of the existence of such different cultures, and that their view of theoretical generalization – necessarily through abstraction – was an idiosyncrasy not shared by the majority of scientific disciplines.

Theoretical statements are, in their essence, *general* statements, i.e. statements that take insights from concrete data (cases, in our culture of interpretation) to a level of plausible extrapolation. Since every case one studies is only a case because it is a case *of something* – an actual and unique instantiation of more generally occurring phenomena – even a single case can be "representative" of millions of other cases. This generalization is always conjectural (something even hardliners from the other camp admit) and demands further testing, in ever more cases. I usually add to this that this method – a scientific one, to be sure – is actually used by your doctor whenever s/he examines you. Your symptoms are always an actual (and unique) instantiation of, say, flu or bronchitis, and your doctor usually concludes the diagnosis on the basis of plausible extrapolation: although s/he can never be 100% sure, the combination of symptoms a, b and c strongly suggests flu. If the prescribed medicine works, this hypothesis is proven correct; if not, the conjectural nature of this exercise is demonstrated. Unless you want to see your doctor as a quack or an alchemist who can’t possibly speak the truth (which would make it highly irrational to go and see him/her when you're ill), it may be safe to see him/her as an inductive scientist working from facts to theory and usually doing a pretty accurate job at that.

People who believe that mathematics and only mathematics equals science, are in actual fact a small but vocal and assertive minority in the scientific community. If they wish to dismiss, say, 70% of what is produced as science as “unscientific”, they do so at their peril (and sound pretty unscientific, even stupid, when they do so). That includes Mr. Popper too. The question “what is science?” is answered in very many forms, as a sovereign rational choice of most of its practitioners. Enforcing the preferences of one member of that community, we heard from Kenneth Arrow, is dictatorial. And since we believe that science is an elementary ingredient of a free and democratic society, and that pluralism in reasoned dialogue, including in science, is such an elementary element as well - we really don’t want that, do we?

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